**Flexible conceptualization in mathematics: Evidence in gesture of multiple and complementary construals of abstract arithmetic**

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Mathematics is notoriously certain and stable, an archetype of abstract thought. But this is an accomplishment, not a given, and actual mathematical practice is flexible, contested, even agonistic (Lakatos, 1976). Indeed, mathematical practice may be a site of pervasive conceptual metaphor and integration (Lakoff and Núñez, 2000; Alexander, 2011). Like many everyday concepts, mathematical notions often invite more than one construal. Lakoff and Núñez (2000), for instance, suggest that we may conceptualize arithmetic metaphorically as MOTION ALONG A PATH or, alternatively, as OBJECT COLLECTION. However, at present there are no experimental studies of the real-time embodied conceptualization of arithmetic. In our study, we asked two questions: Are these two hypothesized construals deployed during real-time reasoning about arithmetic? How stable are these construals, and can individuals be influenced to adopt one or the other? To answer these questions, we looked at participants' reasoning in speech and gesture—their composite conceptualization.

We used a priming paradigm that involved task-irrelevant mental imagery followed by a semi-structured interview about arithmetic. Participants began by completing one of two non-mathematical mental imagery tasks meant to prime relevant image schemas—either Container or Source-Path-Goal. They then answered questions about arithmetic (e.g., "Why is the sum of an odd number and an even number always odd?"), and their responses were video-recorded. We reasoned that, if the conceptualization of arithmetic recycles embodied resources like image schemas, then task-irrelevant mental imagery should prime the associated conceptualization of arithmetic, as revealed in gesture.

Co-speech gesture was segmented into target gestures that co-occurred with talk of arithmetic, and two analysts coded these for handshape (grasping, pointing), handedness (one-, two-handed), and stroke direction. Qualitative and quantitative analyses identified two recurring gesture profiles. "Collecting" gestures were bimanual and used grasping handshapes, and suggested a conceptualization of arithmetic as OBJECT COLLECTION (Fig. 1, top). "Path" gestures were one-handed and used canonical pointing morphology, and suggested a conceptualization as MOTION ALONG A PATH (Fig. 1, bottom). Participants switched fluidly between construals—sometimes even within a single gesture unit—integrating multiple construals of arithmetic during real-time reasoning (cf. Fauconnier and Turner, 2002). Additionally, mental imagery had the predicted effect on subsequent conceptualization as revealed by gesture: Participants who completed the mental imagery task designed to prime a Container schema were significantly more likely to exhibit an OBJECT COLLECTION construal; likewise for Source-Path-Goal imagery and a MOTION ALONG A PATH construal (Fig. 2). The salience of embodied mental resources, therefore, shaped participants' composite conceptualization.

Participants exhibited the kind of flexible and creative conceptual integration that is familiar from studies of discourse or joke comprehension but unexpected in mathematics, a paragon of stability and certainty. In this study, co-speech gesture supplied empirical evidence that the real-time conceptualization of arithmetic is metaphorical and embodied. Moreover, the deployment of these multiple, complementary construals was flexible and sensitive to context. We conclude by suggesting that these shared construals of arithmetic may be a source of imagination and insight, but also a source of inferential stability.

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## Supporting Data

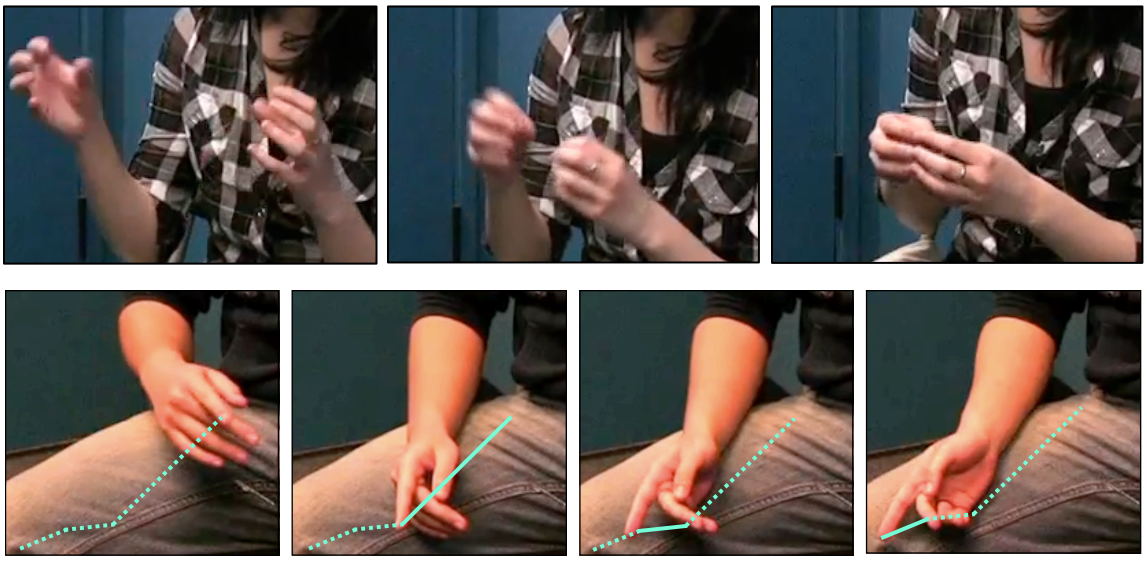

Figure 1: Two participants gesture while reasoning about addition, using a "Collecting" gesture (top panels) and a "Path" gesture (bottom panels).

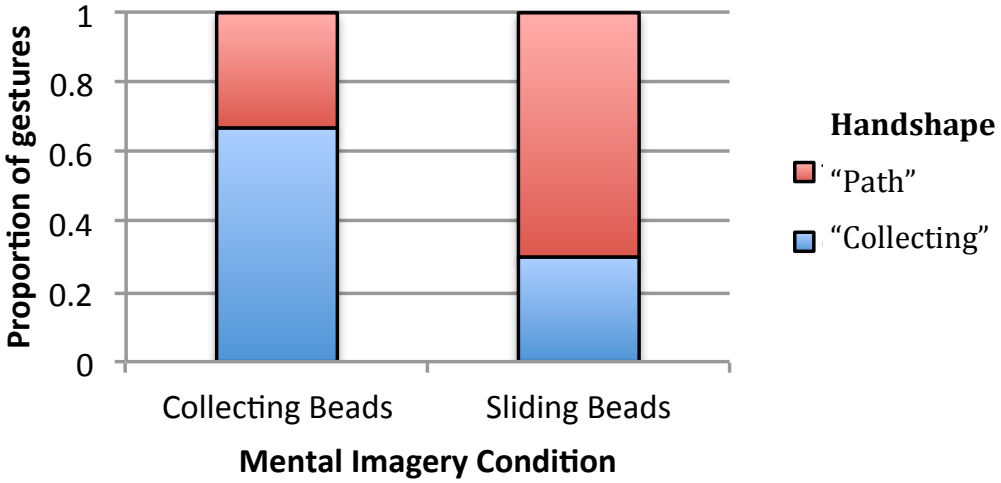


Figure 2: Mental imagery influenced conceptualization, as indexed by handshape (mixed-logit model comparison: $\chi^2$ (1) = 4.07, p < .05) and handedness ($\chi^2$ (1) = 4.16, p < .05).

## References

Alexander, J. (2011). Blending in Mathematics. *Semiotica,* 187, 1-48.
Fauconnier, G. & Turner, M. (2003). *The Way We Think.* New York: Basic Books.
Lakatos, I. (1976). *Proofs and Refutations*. Cambridge: Cambridge University Press
Lakoff, G. & Núñez, R. (2000). *Where Mathematics Comes From.* Basic Books.